Can We Fix It? Adventures into Structural Stability

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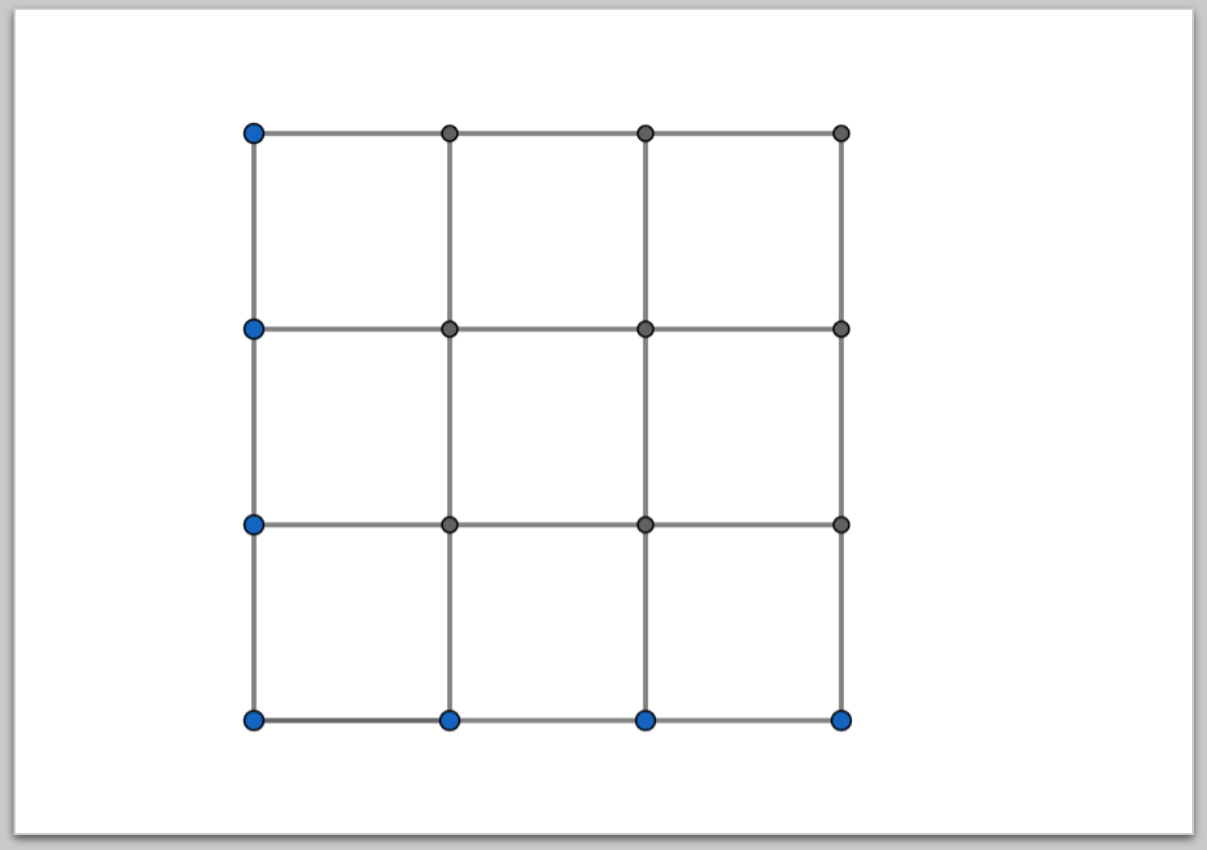
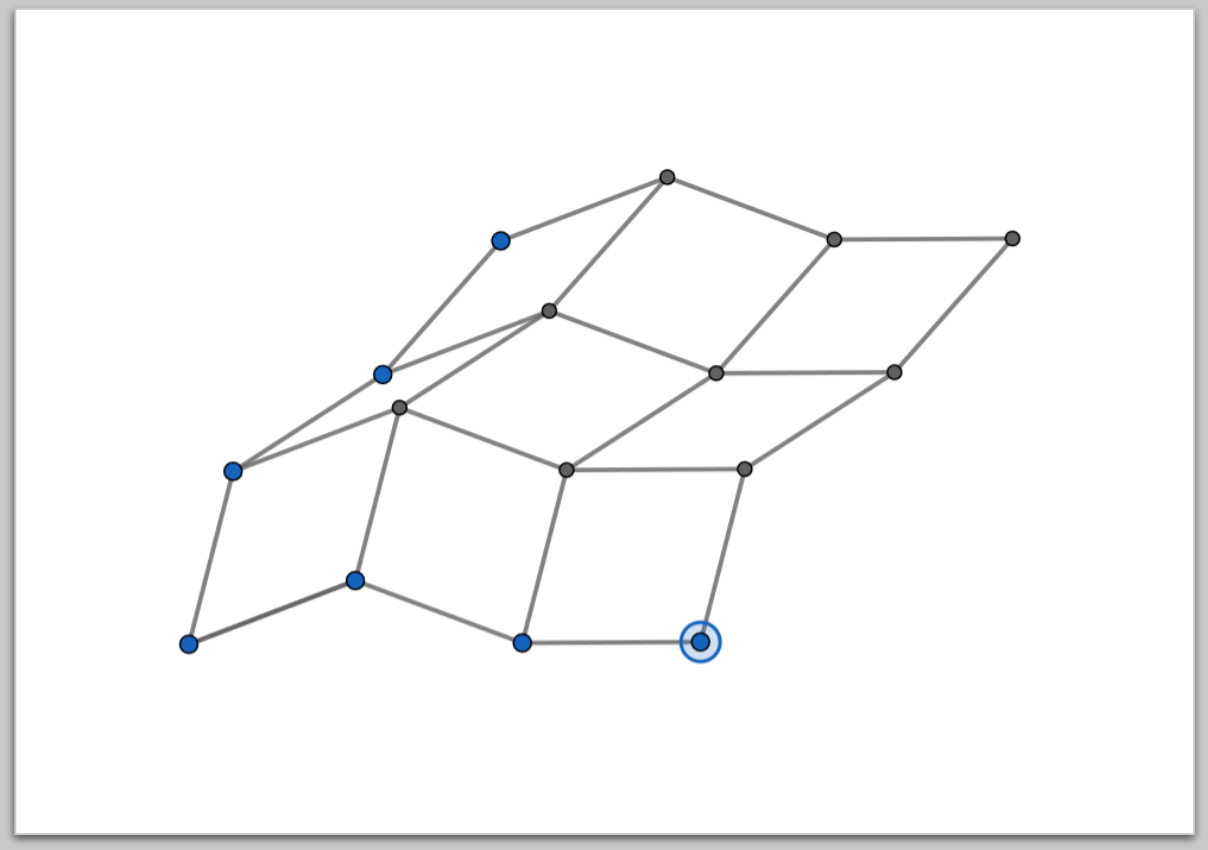
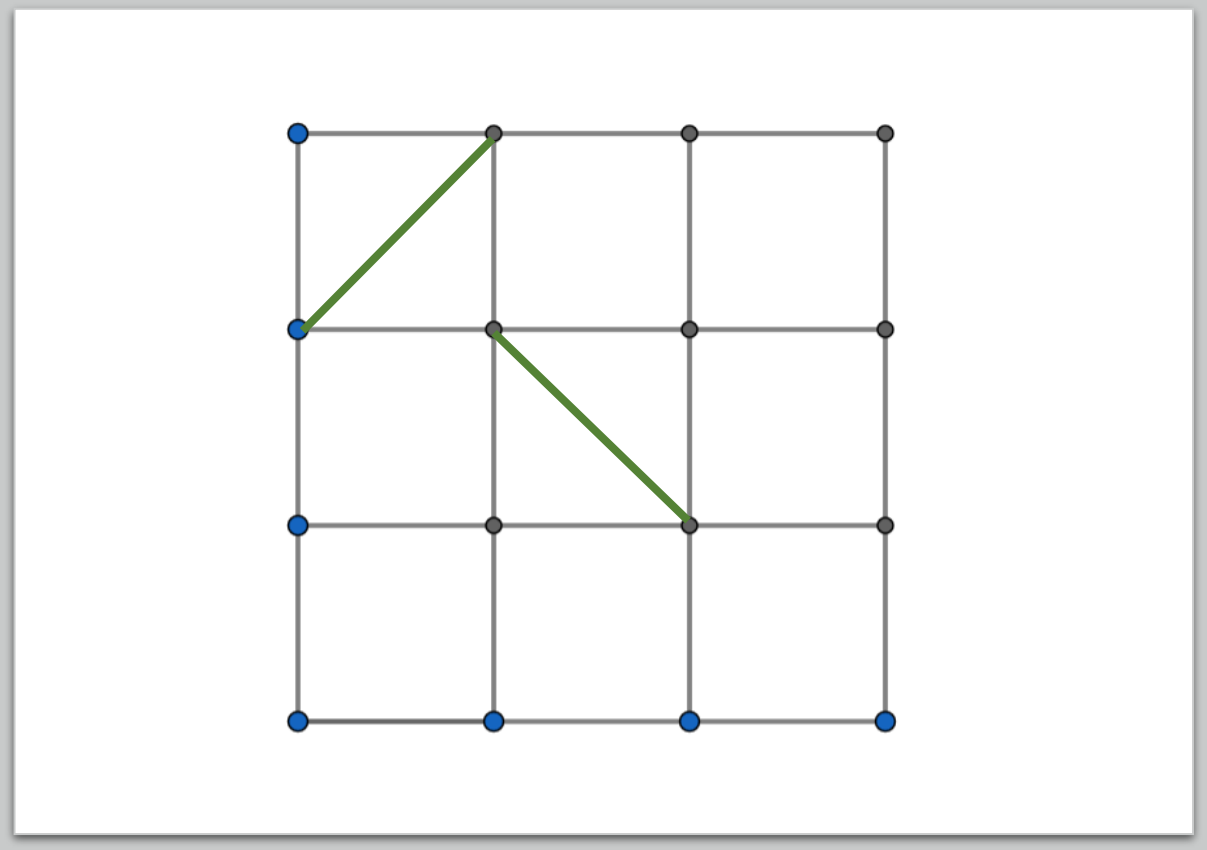
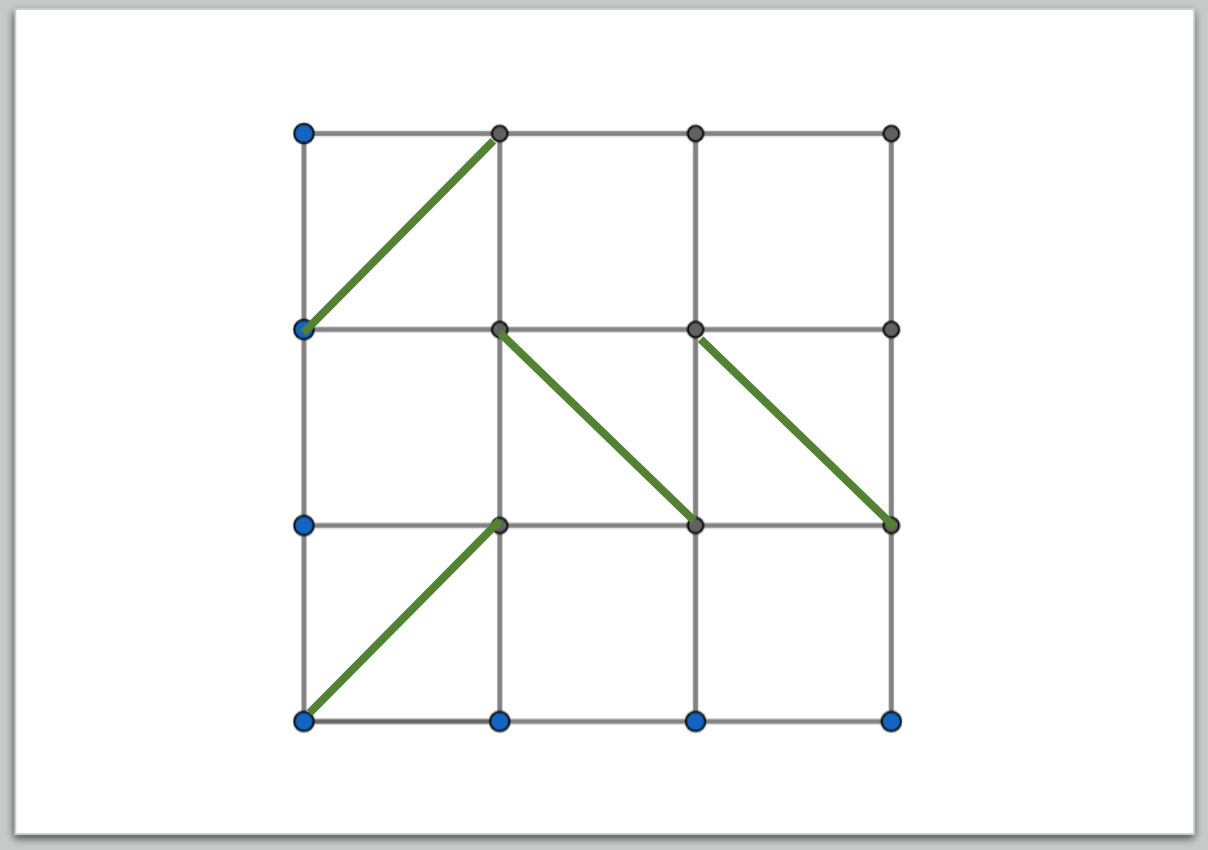
The Problem



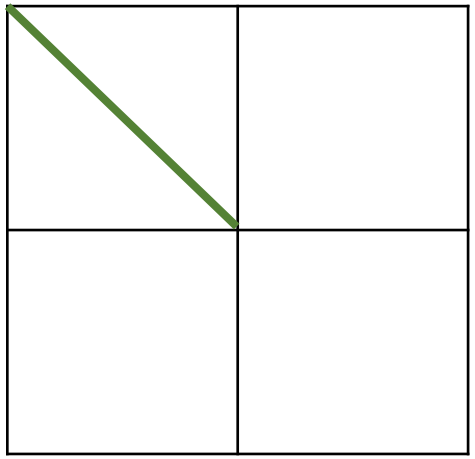


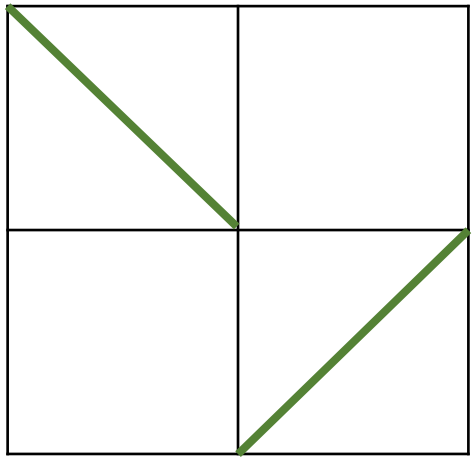
The Solution ??

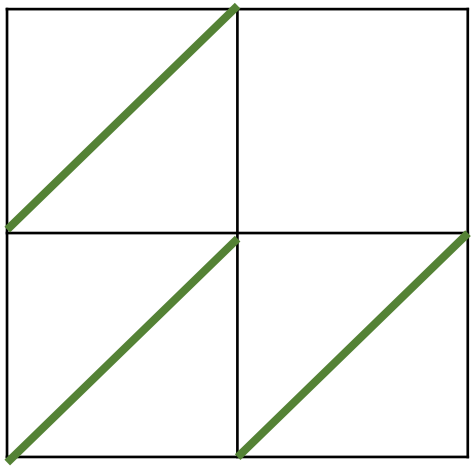




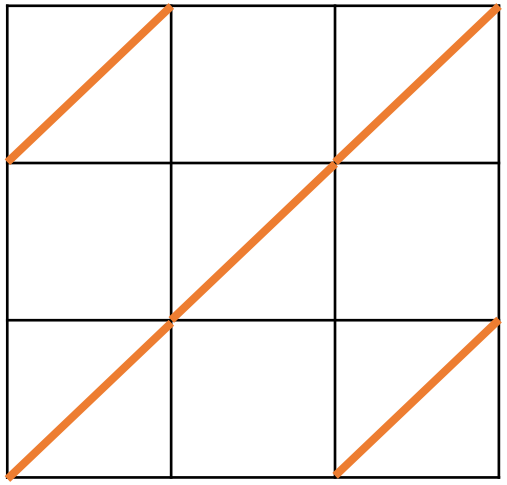
Will it move?



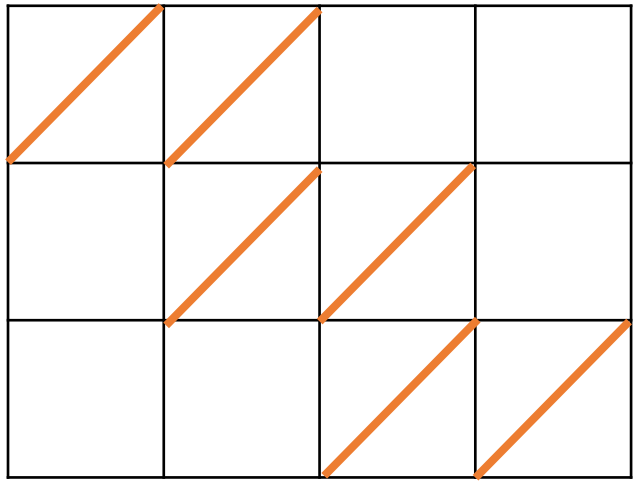




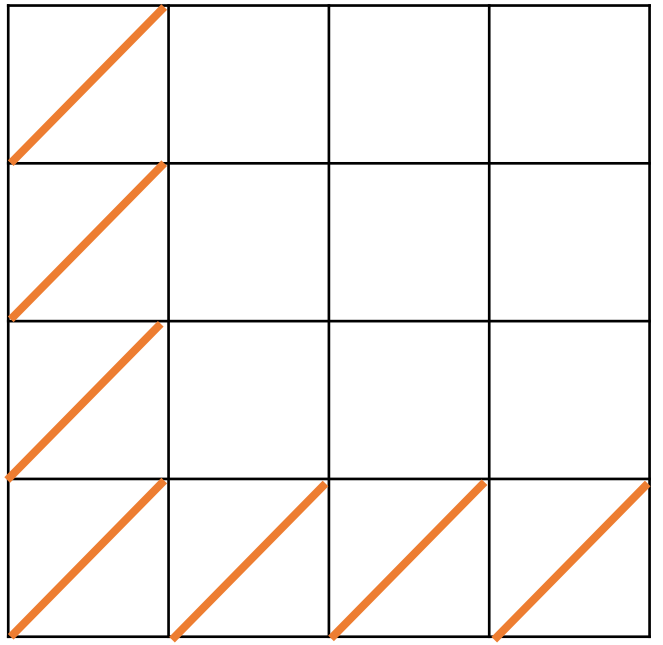
Will it move?



checkerboard grid



staircase grid

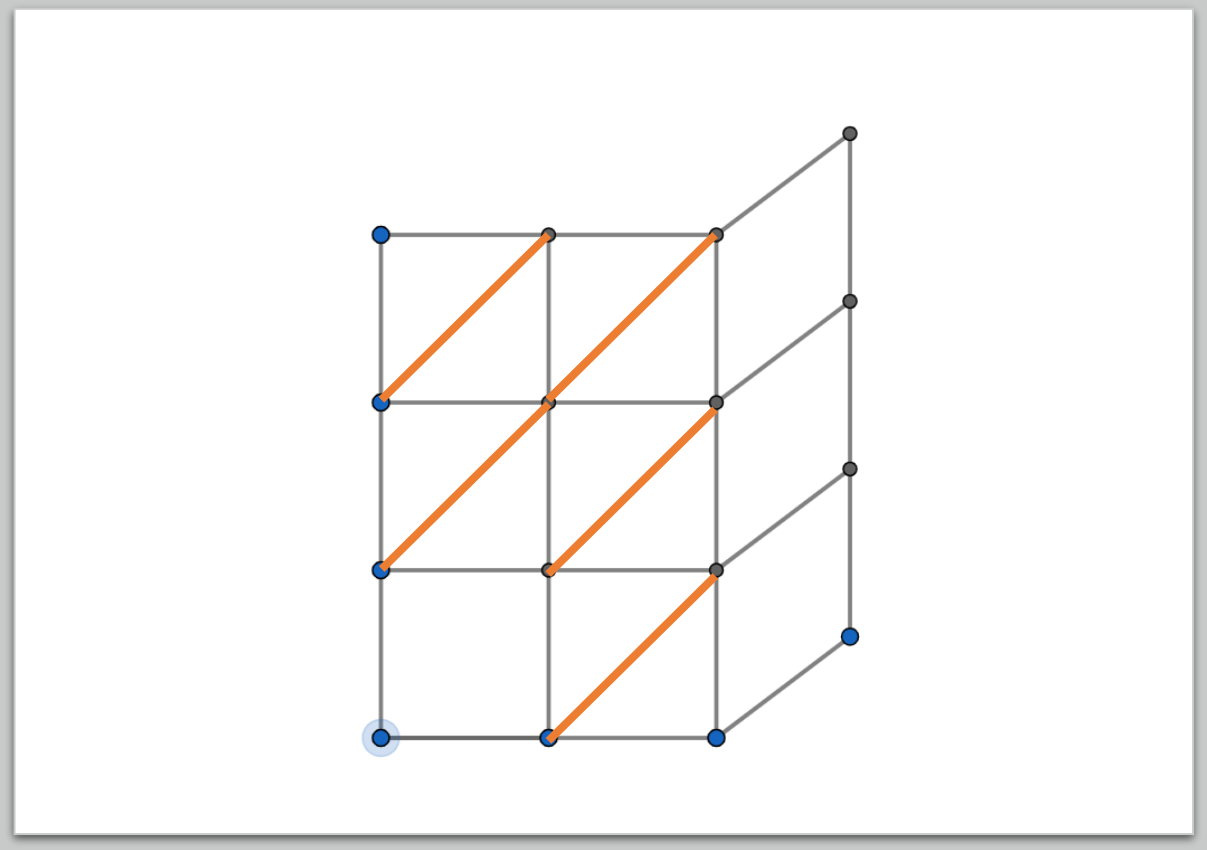


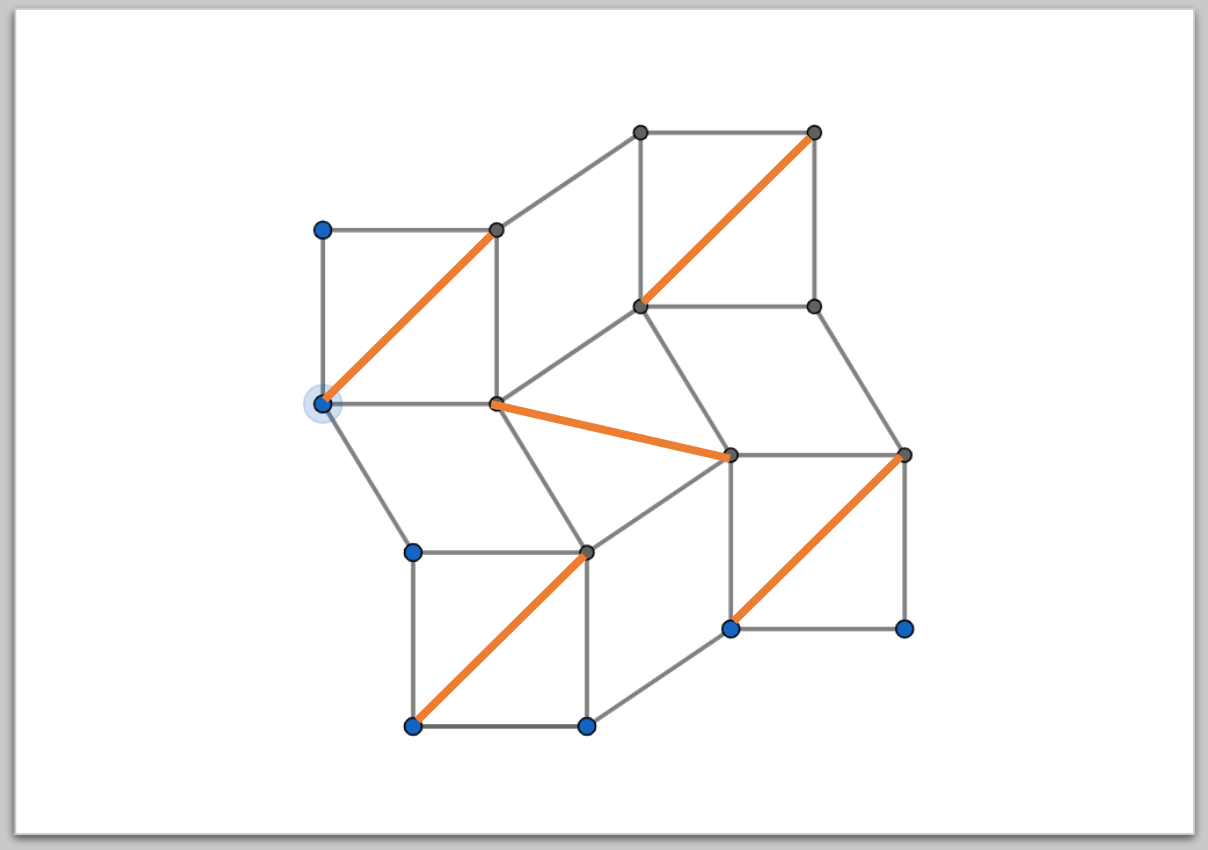
L-grid

Observations

The grid can be deformed if:

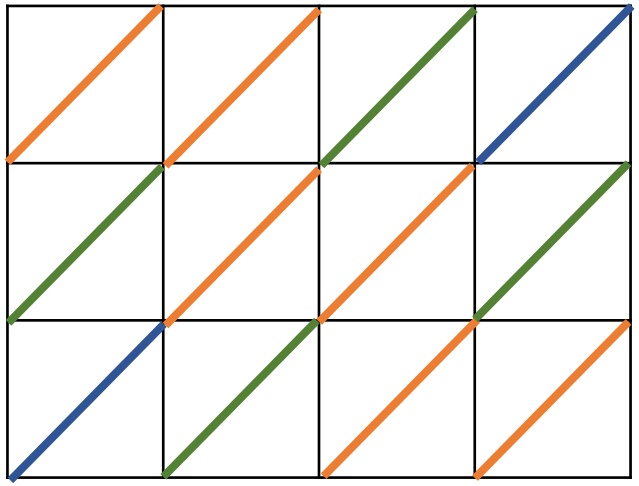
- there is an empty row or column
- There is a brace that is the only one both in its row and its column





Some ideas

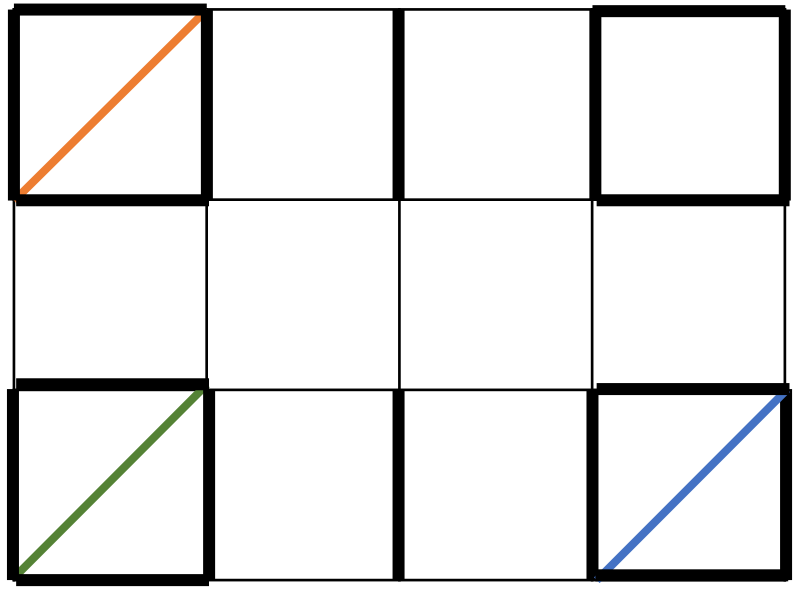
three braces in a 2x2 square – staircase grid example



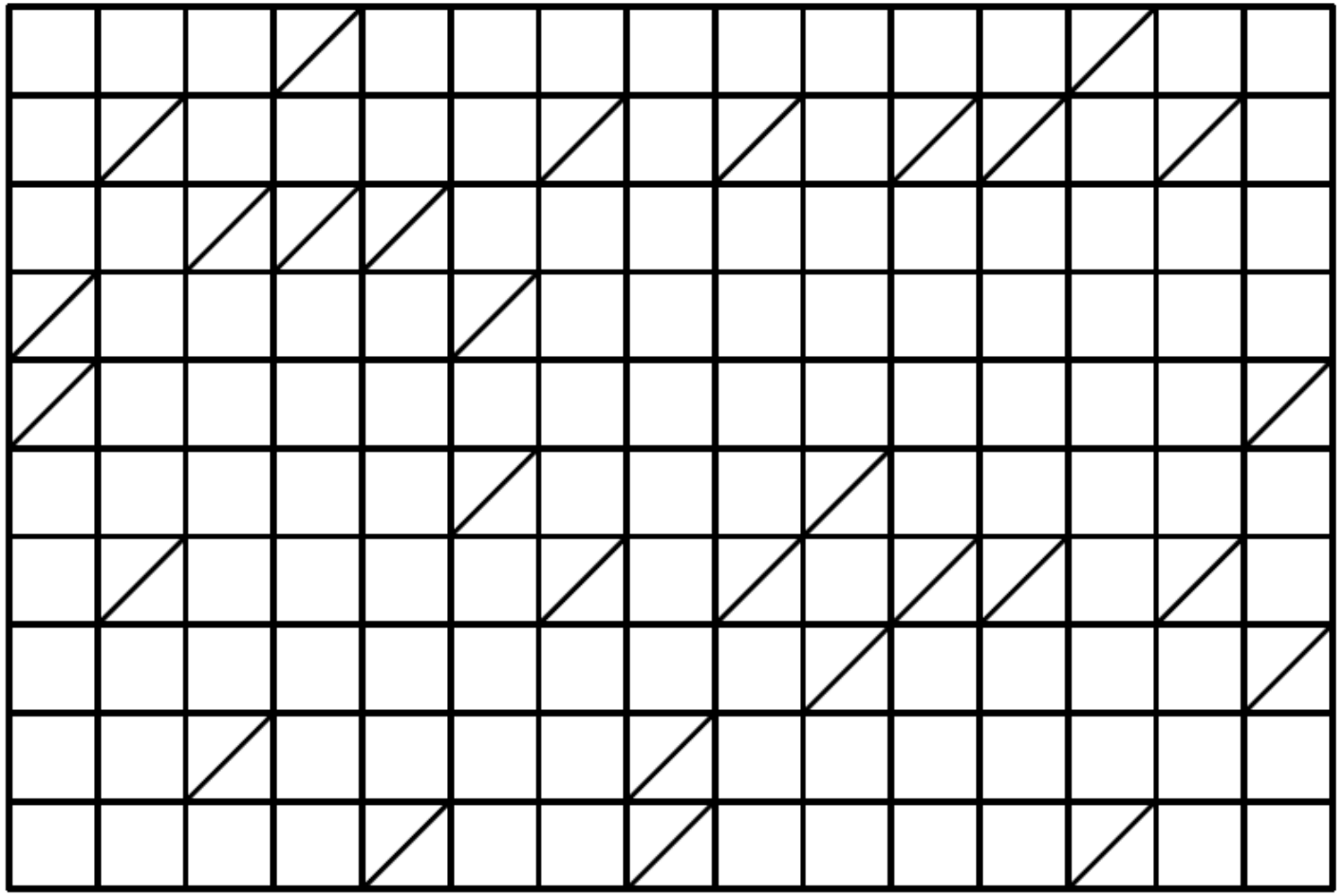
Some more ideas

`horizontal' beams in columns

`vertical' posts in rows

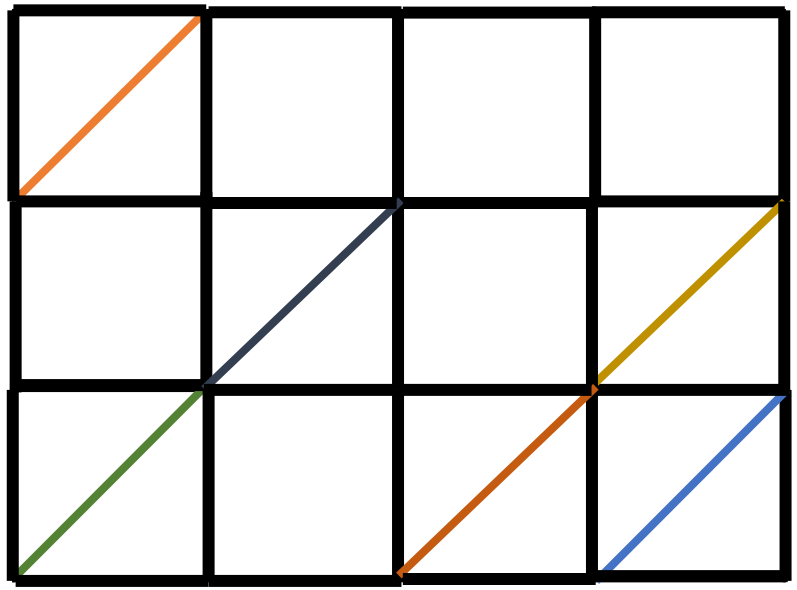


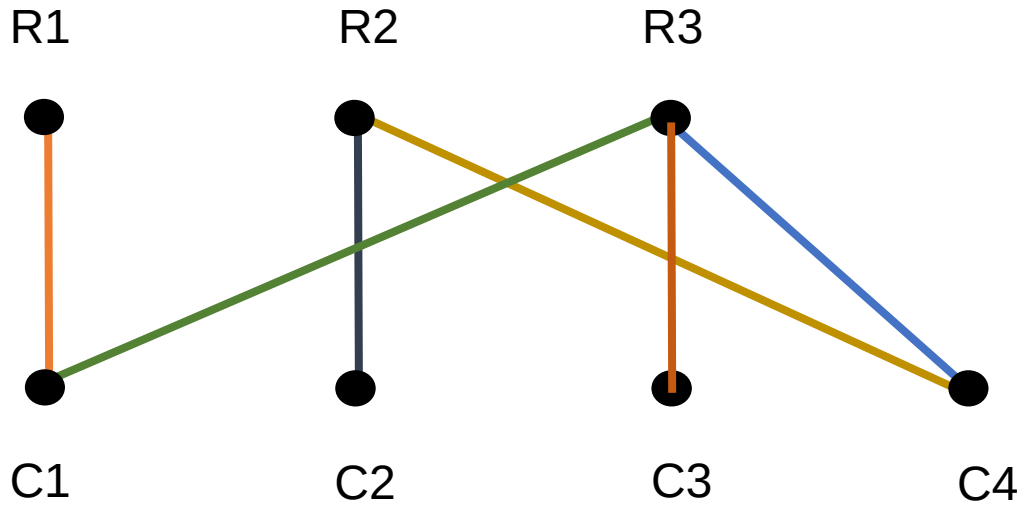
Does it move?



(from FLOPPY GRIDS, Klatt et.al.)

Graph theory to the rescue





A grid is rigid if and only if its grid graph is connected.

Minimal bracing

Subset of the braces that keep the grid rigid but if any of them is removed than the grid can be deformed.

Spanning tree of the connected grid graph has this property.

For an m x n grid any minimal bracing has m+n-1 braces.

Other approaches

Linear algebra

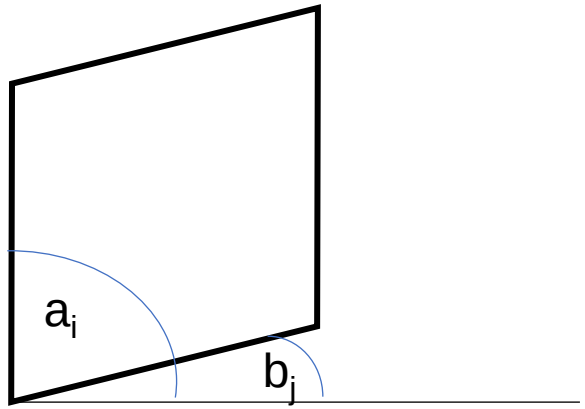
adjacency matrix A for the grid graph of the m x n braced grid

$$R=A+A^2+A^3+ \ldots + A^{(m+n-1)}$$

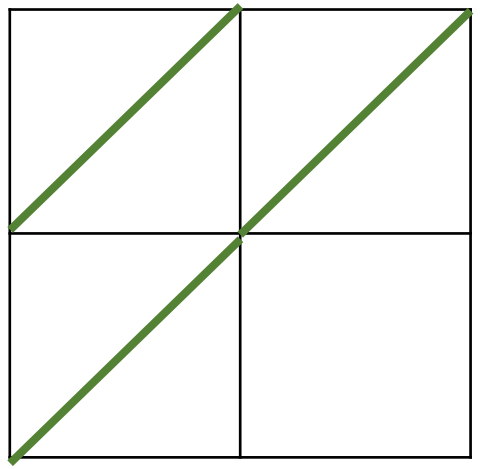
The bracing is rigid if and only if R has no 0 entries.

Angle constraints

square (i,j)



$a_i = b_j + 90^\circ$



$a_1 = 90^\circ$
$a_1 = b_1 + 90^\circ$
$a_1 = b_2 + 90^\circ$
$a_2 = b_1 + 90^\circ$

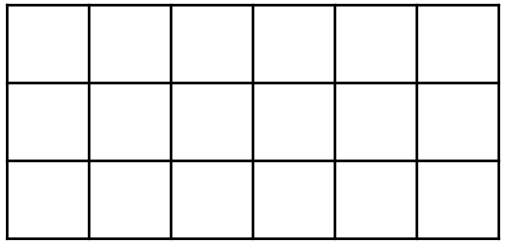
Grid is rigid if $a_i = 90^\circ$ $b_j = 0^\circ$ is the unique solution.

Further explorations

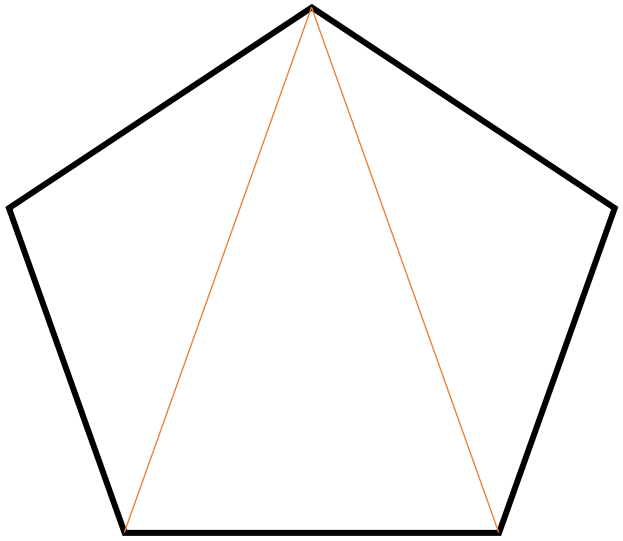
Fault tolerant bracings – what if a brace fails?

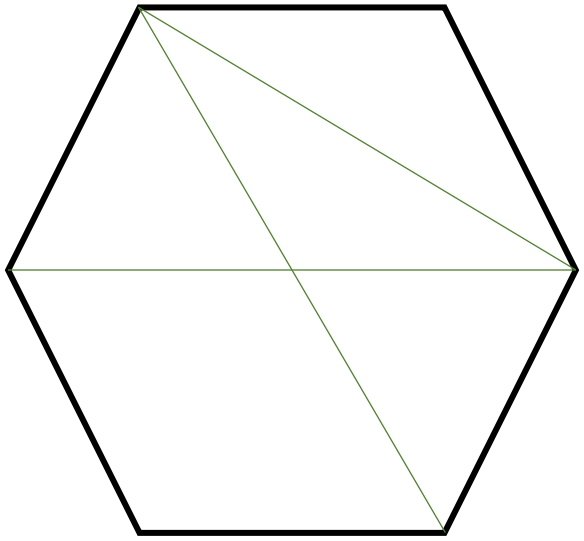
Bracing a polyomino shape

Bracing game

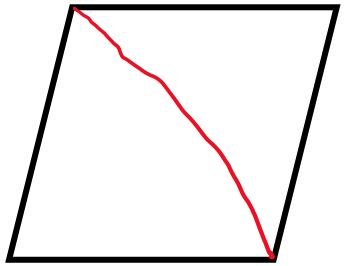


Bracing other shapes

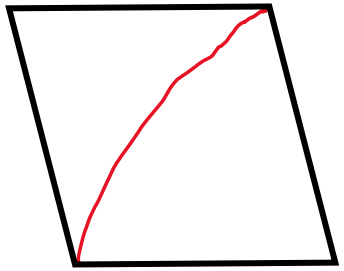




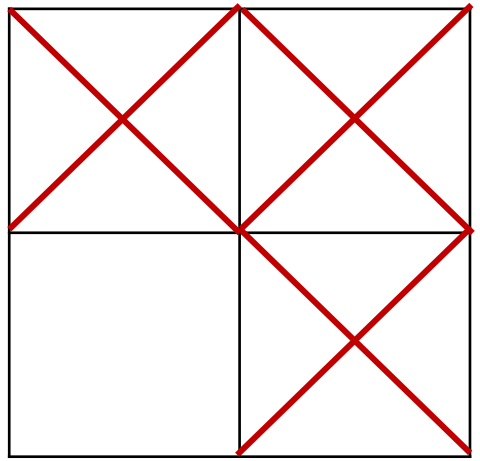
Bracing with cables

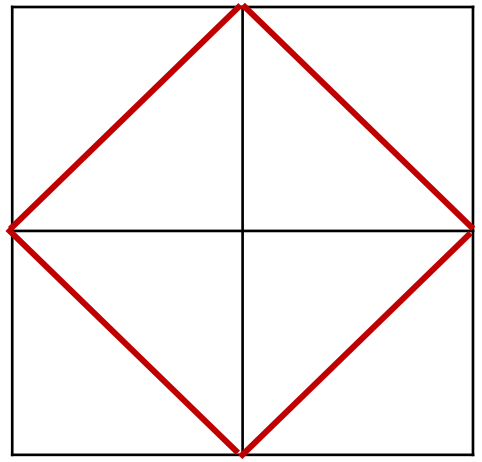


$a_i \leq b_j + 90^\circ$



$a_i \geq b_j + 90^\circ$





Thank you!

References:

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Graphs, Digraphs, and the Rigidity of Grids, Brigitte Servatius, UMAP Journal, 16, 37-63, 1995.

Floppy Grids – Discovering the Mathematics of Grid Bracing, a COMAP FAIM Project by G. Klatt, CC Edwards, S. Heubach and V. Howe

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